Sesión Especial 13

Funciones especiales, polinomios ortogonales y aplicaciones

Organizadores

- Iván Area (Universidad de Vigo)
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Descripción

Las funciones especiales de la física matemática, y entre ellas los polinomios ortogonales, constituyen un destacado objeto de investigación tanto en matemática pura como aplicada, con numerosas aplicaciones en diversas ramas de la ciencia y la tecnología. El objetivo de esta sesión es mostrar algunas de las líneas de investigación actuales englobando tanto aspectos teóricos como aplicados y computacionales. Entre ellos, se abordarán problemas del análisis asintótico (y en particular de polinomios ortogonales) y sus aplicaciones (cuadratura Gaussiana, aproximación asintótica de integrales), diversos temas en la teoría de polinomios ortogonales (ortogonalidad múltiple, ortogonalidad discreta, polinomios de Sobolev, polinomios en el disco), así como algunas aplicaciones a la física, y en concreto a la mecánica cuántica.

Programa

Jueves, 7 de febrero (mañana)

11:30 – 12:00	Nico M. Temme (CWI, Amsterdam) *Asymptotics of the classical orthogonal polynomials and of their zeros from the viewpoint of Gauss quadrature*
12:00 – 12:30	Ester Pérez Sinusía (Universidad de Zaragoza) *Asymptotic approximations of the canonical catastrophe integral*
12:30 – 13:00	Pedro J. Pagola (Universidad Pública de Navarra) *Approximation of special functions: uniform convergent expansions in terms of elementary functions*
13:00 – 13:30	Diego Ruiz-Antolín (Universidad de Cantabria) *Numerical evaluation of discrete Painlevé equations*

Jueves, 7 de febrero (tarde)

15:30 – 16:00	Juan Luis Varona (Universidad de La Rioja) *Bernoulli-Dunkl polynomials*
16:00 – 16:30	Bernardo de la Calle Ysern (Universidad Politécnica de Madrid) *Optimal extensions of optimal quadratures*
16:30 – 17:00	Luis Velázquez Campoy (Universidad de Zaragoza) *The Schur bridge between mathematics and quantum physics*
17:30 – 18:00	Diego Dominici (Johannes Kepler University Linz) *Mehler-Heine type formulas for discrete orthogonal polynomials*
18:00 – 18:30	Amilcar Branquinho (University of Coimbra) *Differential-Difference properties of orthogonal polynomials via Riemann-Hilbert problems*

Viernes, 8 de febrero (mañana)

09:00 – 09:30	Teresa E. Pérez (Universidad de Granada) *Bivariate Sobolev orthogonal polynomials on the disk*
09:30 – 10:00	Judit Mínguez (Universidad de La Rioja) *Fourier series of some Sobolev polynomials*
10:00 – 10:30	Ana Foulquié Moreno (University of Aveiro) *The Riemann Hilbert problem and Generalized Nikishin systems*
10:30 – 11:00	Jorge Arvesú (Universidad Carlos III de Madrid) *On a construction of some rational approximants to* $\zeta(n)$, $n = 2, 3, \ldots$.

Asymptotics of the classical orthogonal polynomials and of their zeros from the viewpoint of Gauss quadrature

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Abstract. Our interest in this topic arose after the publication of a paper by Bogaert (2014) where the Legendre case was considered, which is the simplest example of Gauss quadrature for the classical orthogonal polynomials. We consider the Hermite, Laguerre and Jacobi cases. We show how to use known and new asymptotic expansions of the polynomials to obtain expansions of their zeros. For zeros close to turning points, or other critical points, we need extra expansions of the coefficients in the expansions of the polynomials and the zeros.

References

[1] A. Gil, J. Segura, and N. M. Temme. Non-iterative computation of Gauss-Jacobi quadrature by asymptotic expansions for large degree. Submitted; arXiv:1804.07076 .

[2] A. Gil, J. Segura, and N. M. Temme. Asymptotic approximations to the nodes and weights of Gauss–Hermite and Gauss–Laguerre quadratures. *Stud. Appl. Math.*, 140(3), 2018.

[3] A. Gil, J. Segura, and N. M. Temme. Asymptotic expansions of Jacobi polynomials for large values of β and of their zeros, *SIGMA Symmetry Integrability Geom. Methods Appl.,* 14, 2018.

Joint work with Amparo Gil and Javier Segura, Universidad de Cantabria.
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Asymptotic approximations of the canonical catastrophe integrals

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Abstract. We consider the family of catastrophe integrals $\Psi_K(\mathbf{x}) = \int_{-\infty}^{\infty} \exp(i\Phi_K(t;\mathbf{x}))\,\mathrm{d}t$ with $\Phi_K(t;\mathbf{x}) = t^{K+2} + \sum_{m=1}^{K} x_m t^m$ for large values of one of the variables $|x_m|$ and bounded values of the remaining ones. The integrand of $\Psi_K(\mathbf{x})$ oscillates wildly and the asymptotic analysis is subtle. We use the simplified saddle point method introduced in [2] and derive asymptotic approximations of $\Psi_K(\mathbf{x})$ for large values of the asymptotic variable and fixed values of the other ones. The asymptotic analysis requires the study of different regions separated by the corresponding Stokes lines. Essentially, there are four different asymptotic behaviors according to the even or odd character of the parameters m and K. The accuracy and the asymptotic character of the approximations is illustrated with some numerical experiments (see [1]).

References

[1] C. Ferreira, J. L. López and E. Pérez Sinusía, The asymptotic expansion of the swallowtail integral in the highly oscillatory region, *Appl. Math. Comput.*, **339**(2018), 837—845.

[2] J. L. López, P. Pagola and E. Pérez Sinusía, A systematization of the saddle point method. Application to the Airy and Hankel functions, *J. Math. Anal. Appl.*, **354**(2009), 347–359.

Joint work with Chelo Ferreira and José L. López.

Approximation of special functions: uniform convergent expansions in terms of elementary functions.

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Abstract. In literature we can find a variety of expansions (convergent or not) of dfferent special functions. Usually, these expansions are not valid at the same time for small and large values of its variable z. In this work, we derive new expansions of several special functions in terms of elementary functions of its variable z that converge uniformly in different regions, bounded or unbounded, of the complex plane. We give either, explicit formulas for the coefficients of the expansions. The starting point of the analysis is a convenient integral representation of the special function. The key point of the technic is the approximation of an appropriate factor of the integrand that not depend on z. After the interchange of series and integral this fact, the independence of z, translates into a remainder that may be bounded independently of z. Finally, we show the accuracy of the approximations by means of several numerical experiments.

Joint work with Blanca Bujanda, José L. López, Dmitrii B.Karp

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Numerical evaluation of discrete Painlevé equations

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Abstract. Many cases of discrete Painlevé equations arise from the study of the coefficients of the recurrence relations of certain orthogonal polynomials associated to semi-classical weights. A naive approach to the evaluation of the solutions of these equations could be the direct evaluation of the nonlinear recurrence relations that define them. However, there are multiple cases like the alternative discrete Painlevé I (alt-dP_I) or the discrete Painlevé IV (dP_{IV}) where the direct evaluation in both directions of the recurrence can be unstable for the nonnegative case. But there are also certain cases of discrete forms of Painlevé III that don't have this instability. We observe that the unstable solutions have as an initial point recesive solutions of certain linear differential equations like the case of alt-dP_I for which the initial point is a combination of Airy functions Ai(x). We are interested in both explaining the cause of these instabilities and describing numerical and asymptotic methods to evaluate the nonnegative solutions of these recurrence relations when the direct evaluation is unstable.

Joint work with Alfredo Deaño Cabrera

Bernoulli-Dunkl polynomials

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Abstract. In the talk we will explain how to define the Bernoulli polynomials (or the Euler polynomials) in the Dunkl context, as well as some of the properties and applications that we can obtain. This is a new and unexplored context for Appell sequences, where we have already written the papers [1, 2, 3], and we are sure that a lot of future work is yet possible.

References

[1] Ó. Ciaurri, A. Durán, M. Pérez and J. L. Varona, Bernoulli-Dunkl and Apostol-Euler-Dunkl polynomials with applications to series involving zeros of Bessel functions, *J. Approx. Theory* **235** (2018), 20–45.

[2] Ó. Ciaurri, J. Mínguez Ceniceros and J. L. Varona, Bernoulli-Dunkl and Euler-Dunkl polynomials and their generalizations, preprint.

[3] A. Durán, M. Pérez and J. L. Varona, Fourier-Dunkl system of the second kind and Euler-Dunkl polynomials, preprint.

The content of the conference is the result of works in collaboration with Óscar Ciaurri (Universidad de La Rioja), Antonio Durán (Universidad de Sevilla), Judit Mínguez (Universidad de La Rioja) and Mario Pérez (Universidad de Zaragoza), cited in the references.

Optimal extensions of optimal quadratures

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Abstract. The Kronrod extension of the Gaussian quadrature formula arose in the sixties of the last century as an efficient method to compute approximated values of an integral together with an estimation of the error committed. When the nodes are real and the quadrature weights are positive, the Gauss-Kronrod rule can be computed efficiently and is frequently used in packages for automatic integration [1]. The fact, on the other hand, that these properties do not hold for all weight functions has led to consider sub-optimal Gaussian extensions. In this talk we will review the most salient aspects of the Gauss-Kronrod rule as well as some recent related results [2, 3].

References

[1] D. Calvetti, G. H. Golub, W. B. Gragg, and L. Reichel, Computation of Gauss-Kronrod quadrature rules, *Math. Comp.* **69** (2000), 1035–1052.

[2] B. de la Calle Ysern, Optimal extension of the Szegő quadrature, *IMA J. Numer. Anal.* **35** (2015), 722–748.

[3] B. de la Calle Ysern and M. M. Spalević, Modified Stieltjes polynomials and Gauss–Kronrod quadrature rules, *Numer. Math.* **138** (2018), 1–35.

The Schur bridge between mathematics and quantum physics

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Abstract. Several problems coming from apparently unrelated areas of mathematics and quantum theory have revealed very recently a close connection. At the heart of these unexpected links is the notion of Schur function, one of the gems resulting from the interplay between harmonic analysis and complex variables, bequeathed to us by Issai Schur, a very original worker who left his mark all over mathematics. It turns out that very different mathematical and physical problems, formulated in the common language of Schur functions, feed into each other giving rise to a symbiotic spiral which ends in the solution to all of them. The talk will present these results, emphasizing the essential role that the exchange of ideas among different fields may play in the solution of multiple problems. The alluded results touch areas of harmonic analysis, orthogonal polynomial theory, random walks, as well as their quantum version, known as quantum walks.

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Mehler-Heine type formulas for discrete orthogonal polynomials

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Abstract. We derive Mehler–Heine type asymptotic expansions for discrete orthogonal polynomials. These formulas provide good approximations for the polynomials in the neighborhood of $x = 0$, and determine the asymptotic limit of their zeros as the degree n goes to infinity.

References

[1] D. Dominici. Mehler-Heine type formulas for Charlier and Meixner polynomials II. Higher order terms. *J. Class. Anal.*, 12(1):9–13, 2018.

[2] D. Dominici. Mehler-Heine type formulas for Charlier and Meixner polynomials. *Ramanujan J.*, 39(2):271–289, 2016.

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Differential-Difference properties of orthogonal polynomials via Riemann-Hilbert problems

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Abstract. In this talk we survey different aspects of the orthogonal polynomials theory. Due to the ubiquity nature of orthogonal polynomials in modern science, their study has been performed from very many different points of view. Indeed, they can be considered from a purely algebraic point of view, from that of approximation theory, and in the context of modern measure theory and functional analysis. Matrix Riemann-Hilbert problem for orthogonal polynomials is a powerful tool connecting the two main areas of the orthogonal polynomials theory. In this talk we intend to introduce the main objects in the theory of orthogonal polynomials and also to present this Riemann-Hilbert problem.

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Bivariate Sobolev orthogonal polynomials on the disk

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Abstract. Sobolev orthogonal polynomials in two variables are defined via inner products obtained by adding to a standard measure another term involving derivation tools such as gradients, normal derivatives, partial derivatives, etc. Such a kind of orthogonal polynomials appears for the first time in 2008 in a problem related to dwell time for polishing tools in fabricating optical surfaces. In this talk, we study the particular case when both measures are the classical measure on the unit disk, and we will show new properties for orthogonal polynomials associated with this kind of Sobolev inner products.

Fourier series of some Sobolev polynomials

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Abstract. The study of Sobolev orthogonal polynomials has attracted the interest of many researchers in the last years as we can see in [1]. The main target of our work is the study of the convergence of the Fourier series in terms of Sobolev orthonormal polynomials. Pollard in [2] and [3] studied the uniform boundedness of the partial sum operators in L^p for Gegenbauer and Jacobi polynomials, and as a consequence, the convergence of the Fourier series in L^p. In this talk, we will show a complete characterization of the uniform boundedness of the partial sums in some Sobolev cases. Moreover, we will study the completion of our Sobolev spaces to show that uniform boundedness and convergence of the series could be no equivalent.

References

[1] F. Marcellán and Y. Xu, On Sobolev orthogonal polynomials, *Expo. Math.* **33** (2015), 308–352.

[2] H. Pollard, The mean convergence of orthogonal series. II, Trans. Amer. Math. Soc. **63** (1948), 355–367.

[3] H. Pollard, The mean convergence of orthogonal series. III, Duke Math. J. **16** (1949), 189–191.

Joint work with O. Ciaurri and J.M. Rodríguez.

The Riemann Hilbert problem and Generalized Nikishin systems

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Abstract. Hermite Padé approximants for systems of Markov functions appear in the proof of the transcendece of e by Hermite. The so called Angelesco and Nikishin systems are examples of these systems, extensively studied. Gonchar Rakhmanov and Sorokin introduce in [2] systems of Markov functions described by a graph, the Generalized Nikishin systems. We present the Riemann-Hilbert problem that has as a solution the multiple orthogonal polynomials with respect to the measures generated by these systems and explore the recurrence relation and Christoffel–Darboux formula that these multiple orthogonal polynomials satisfy.

References

[1] A. Foulquié Moreno, Riemann-Hilbert problem for a generalized Nikishin system, in ”Diference equations, Special Functions and Orthogonal Polynomials” (S. Elaydi et al., eds), World Scientific, Hackensack, N. J., 2007, pp. 412-421.

[2] A.A. Gonchar, E. A. Rakhmanov, and V.N. Sorokin, Hermite Padé approximants for systems of Markov-type functions, Acad. Sci. Sb. Math. **188** (1997), 671-696.

[3] W Van Assche, J. S. Geronimo and A. B. Kuijlaars, Riemann-Hilbert problems for multiple orthogonal polynomials, in: J. Bustoz et al. (Eds), Special Functions 2000: Current Perspective and Future Directions, Kluwer, Dordrecht, 2001, pp. 23-59.

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On a construction of some rational approximants to $\zeta(n)$, $n = 2, 3, \ldots$.

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Abstract. An approach based on a Riemann-Hilbert problem associated to a simultaneous rational approximation problem involving the Riemann zeta function will be discussed. As a consequence of the proposed approach infinitely many rational approximants (Diophantine approximations) to $\zeta(3)$ proving its irrationality can be generated. Moreover, this approach unifies the most famous 'independent' proofs of the irrationality of $\zeta(3)$. In addition, rational approximants for $\zeta(5), \zeta(7), \ldots, \zeta(41), \ldots$ will be presented.

Joint work with A. Soria-Lorente (Universidad de Granma, Cuba); in an early stage with P. Deift (Courant Institute of Mathematical Sciences, NYU) and J. Geronimo (School of Mathematics, GaTech).